Impact of Mistiming on the Achievable Information Rate of Rake Receivers in DS-UWB Systems

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Abstract—In this paper, we investigate the impact of mistiming on the performance of Rake receivers in direct-sequence ultra-wideband (DS-UWB) systems from the perspective of the achievable information rate. A generalized expression for the performance degradation due to mistiming is derived. Monte Carlo simulations based on this expression are then conducted, which demonstrate that the performance loss has little relationship with the target achievable information rate, but varies significantly with the system bandwidth and the multipath diversity order, which reflects design trade-offs among the system timing requirement, the bandwidth and the implementation complexity. In addition, the performance degradations of Rake receivers with different multipath component selection schemes and combining techniques are compared. Among these receivers, the widely used maximal ratio combining (MRC) selective-Rake (S-Rake) suffers the largest performance loss in the presence of mistiming.

I. INTRODUCTION

Ultra-wideband (UWB) is promising for wireless high rate and short range communications [1]. Direct-sequence UWB (DS-UWB) [2] has received considerable interest due to its fine properties of coherent processing of the occupied bandwidth and the widest contiguous bandwidth [3].

To exploit the ample multipath diversity, the Rake reception is widely employed in DS-UWB systems [4]. Various types of Rake receivers, like selective Rake (S-Rake) and partial Rake (P-Rake), are proposed recently [5]. However, Rake receivers have stringent requirements for timing accuracy [6]. In practical DS-UWB systems, mistiming due to acquisition and tracking errors is inevitable, thus its effects on the performance degradation is worthy of investigation. Several studies have explored this issue in UWB systems [7]-[9]. In [7], it is shown that the system throughput degrades significantly with relatively modest increase in timing errors over additive white Gaussian noise (AWGN) channels. In [8] and [9], the authors analyze the bit error rate (BER) degradation induced by mistiming for both fixed and random channels in UWB systems based on Rake reception. Compared with throughput and BER, the achievable information rate, which identifies the maximum mutual information between the input and output of one communication system, is a more fundamental

measurement for system performance, and is also a subject of continuing research in UWB systems [10]. To the best of the authors' knowledge, the effect of mistiming on the achievable information rate of Rake receivers in DS-UWB systems has not been investigated yet to date.

In this paper, a systematic approach is presented to evaluate the impact of imperfect timing on Rake receivers in DS-UWB systems from the perspective of the achievable information rate. The influence of key system parameters on the performance of various types of Rake receivers is also investigated. In our analysis, a two-step procedure is adopted. First, a generalized expression of the system performance degradation due to timing mismatch is derived. Then based on this expression, the numerical results are obtained by averaging over a sufficiently large number of channel realizations. The major contributions of this paper lie in the following: (1) As for the widely used maximal ratio combining (MRC) S-Rake receiver, we observe that the performance degradation has little relationship with the target information rate, but varies significantly with the occupied bandwidth and the diversity order, which reflects design trade-offs among the system timing requirement, the bandwidth and the implementation complexity. (2) The performance degradation of various Rake receivers, including MRC S-Rake, MRC P-Rake and equal gain combining (EGC) P-Rake, are compared. Such comparisons shed light on the robustness of various multipath component selection schemes and combining techniques to the variation of system parameters in the presence of mistiming.

This paper is organized as follows: Section II describes the DS-UWB system model. In Section III, from the perspective of the achievable information rate, we derive a generalized expression for the system performance degradation induced by timing mismatch. In Section IV, Monte Carlo simulations based on the analytic derivation are conducted to investigate the influence of some key parameters on the system performance and compare the performance degradation of various Rake receivers under mistiming. Section V draws conclusions.

II. SYSTEM MODELS

Motivated by current DS-UWB system implementations, we confine our discussions to binary phase-shift keying (BPSK)
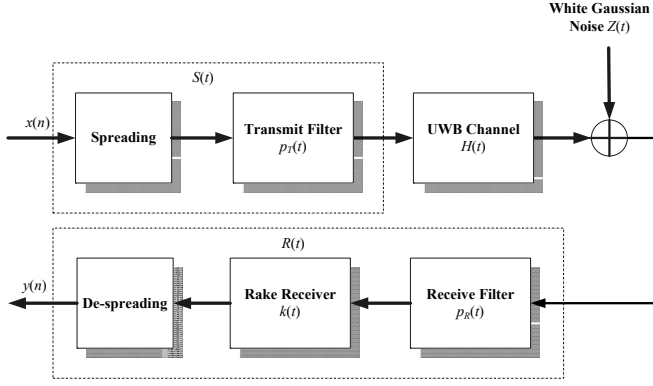
Fig. 1. Block diagrams for the Rake receiver with pulse shaping filters in DS-UWB systems

modulation. The equivalent complex-valued system baseband model considered throughout this paper is shown in Fig.1.

A. Transmitter Model

In DS-UWB systems, the random source symbol is spreaded and then modulated with chip pulse $p_T(t)$. For each symbol, the transmitted waveform is defined as

$$S(t) = \sum_{n=0}^{N-1} c[n]p_T(t - nT_c) \tag{1}$$

where $c[n]$ denotes the n-th chip of the spreading code of length N, and T_c is the chip duration.

B. UWB Channel Model

In this model, the IEEE802.15.3a UWB indoor channel for wireless personal area networks (WPAN) is considered [11]. It states that the magnitude of channel amplitude better agrees with the lognormal distribution, corresponding to the shadowing phenomenon which arises from a more serious fluctuation than ordinary fading in the impulse response [12]. In addition, multipath arrivals are grouped into two categories: cluster arrivals, and ray arrivals within each cluster. The channel impulse response is defined as:

$$H(t) = X \sum_{l=0}^{L-1} \sum_{k=0}^{K-1} \alpha_{k,l}\delta(t - T_l - \tau_{k,l}) \tag{2}$$

where $\delta(t)$ represents the impulse function, X stands for the log-normal shadowing, $\alpha_{k,l}$ denotes the multipath gain coefficient, T_l is the delay of l-th cluster and $\tau_{k,l}$ is the delay of the k-th multipath component relative to the l-th cluster arrival time (T_l). By definition, we have $\tau_{0,l} = 0$ for $l \in \{0, 1, ..., L-1\}$.

C. Reception Model

At the receiver, $p_R(t)$ is matched to the impulse response of the transmit filter $p_T(t)$. In current DS-UWB systems, the raised cosine filter is commonly employed as the pulse shaping filter, which is always achieved by implementing root raised cosine filters as the transmit and receive filters [13]. Therefore,

throughout the rest of this paper, we will consider that the overall impulse response $p(t) = p_T(t) * p_R(t)$ corresponds to a raised cosine filter, which means that $p(t)$ can be written as

$$p(t) = p_T(t) * p_R(t) = \frac{\sin(\pi t/T)}{\pi t/T} \frac{\cos(\alpha \pi t/T)}{1 - 4\alpha^2 t^2/T^2} \tag{3}$$

where α is the roll-off factor, which represents the excess bandwidth of the filter and is a real number ranging from 0 to 1.

In this reception model, we assume that the perfect channel information of the UWB channel is available at the Rake receiver. The impulse response of the Rake receiver can be written as

$$k(t) = \sum_{j=1}^{J} w_j\sigma(t - t_j) \tag{4}$$

where J stands for the Rake diversity orders i.e. finger numbers, w_j is the path weights, and t_j denotes the path delay satisfying $t_j < t_{j+1}$. In P-Rake receiver, the first arrival J multipath components are combined, while the S-Rake receiver selects out the most J strongest multipath components and then combines them together. The Rake weights w_j are selected according to different linear combining techniques. For MRC, $w_j = a_j^*$, while for EGC, $w_j = sign(\alpha_j)$, where a_j denotes the actual path amplitude, $(.)^*$ represents complex conjugation, and $sign(.)$ is the signum function.

Finally, de-spreading is performed to get the symbol-level estimation of transmitted data $y(n)$. The whole DS-UWB receiver, including the matched filter $p_R(t)$, the Rake receiver $k(t)$ and the de-spreading operation, can be expressed as

$$R(t) = \sum_{j=1}^{J} \sum_{n=0}^{N-1} c[n]w_j p_T(t - nT_c - t_j)$$
$$= \sum_{j=1}^{J} w_j S(t - t_j) \tag{5}$$

Finally, let the impulse response given by $S(t) * H(t) * R(t)$ be denoted by $g(t)$, and its symbol-sampled version be $g(n)$. Then we can write $y(n)$ as

$$y(n) = \sum_{k} x(n - k)g(k) + w(n) \tag{6}$$

where $w(n)$ represents the noise component at the Rake output. In (6), $w(n)$ is the symbol-sampled version of $w(t)$, and

$$w(t) = Z(t) * R(t) \tag{7}$$

where $Z(t)$ represents the channel noise which is modeled as AWGN.

III. PERFORMANCE ANALYSIS UNDER MISTIMING

In this section, the performance degradation induced by timing mismatch for Rake receivers in DS-UWB systems is derived in terms of the achievable information rate.

In the DS-UWB system model, when the length of spreading code N is sufficiently large, the autocorrelation property

of spreding code is ideal. Hence the equivalent channel response between the source symbols $x(n)$ and the symbol-level received data $y(n)$ can be simplified to

$$h(t) = p_T(t) * H(t) * p_R(t) * k(t) \qquad (8)$$

Its symbol-sampled version is denoted as $h(k)$.

When mistiming is caused by acquisition or tracking errors in the DS-UWB receiver, the branch delays in the Rake receiver get inaccurate. Denote this timing mismatch in all branches as

$$\Delta t := t'_j - t_j (\forall j \in \{1, 2...J\}) \qquad (9)$$

where t_j is the actual path delay for path j, and t'_j is the estimated path delay. In this case, the impulse response of the Rake receiver is given by

$$\begin{aligned} k'(t) &= \sum_{j=1}^{J} w_j \sigma(t - t'_j) \\ &= \sum_{j=1}^{J} w_j \sigma(t - t_j - \Delta t) \\ &= k(t - \Delta t) \end{aligned} \qquad (10)$$

The corresponding equivalent channel with timing errors is then expressed as

$$\begin{aligned} f(t) &= p_T(t) * H(t) * p_R(t) * k(t - \Delta t) \\ &= h(t - \Delta t) \end{aligned} \qquad (11)$$

Its symbol-sampled version is written as $f(k)$. This timing mismatch will result in performance loss in DS-UWB systems.

It is also worthwhile noting that, when the spreading code attains ideal autocorrelation property and the amplitude modulation schemes, especially those with bipolar modulation, e.g. BPSK, are employed, the inter-chip interference (ICI) can be reduced to a negligible order [14]. Furthermore, to simplify the analysis, we also assume the excess multipath delay is smaller than several symobl periods, therefore the effect of inter-symbol interference (ISI) on the DS-UWB system is also limited. In this case, where the ICI and ISI are ignorable, the noise component at the Rake output $w(n)$ can be regarded as AWGN [14].

In the system model described in section II, the source data are constrained to be independent and identically distributed (i.i.d.). The system achievable information rate, which corresponds to the maximum mutual information between the input $x(n)$ and the output $y(n)$, is given by [15] [1]

$$C = \frac{1}{4\pi} \int_{-\pi}^{\pi} \log_2[1 + 2\frac{E_s}{N_0}|H(e^{j\theta})|^2]d\theta \qquad (12)$$

where E_s is the symbol energy, and $H(e^{j\theta})$ is the Fourier transform of the equivalent channel impulse response. In the

[1]When the white Gaussian noise at the Rake output is not valid, i.e. in the case of colored Gaussian noise, the calculation of achievable information rate can be performed by using the method of water pouring [16].
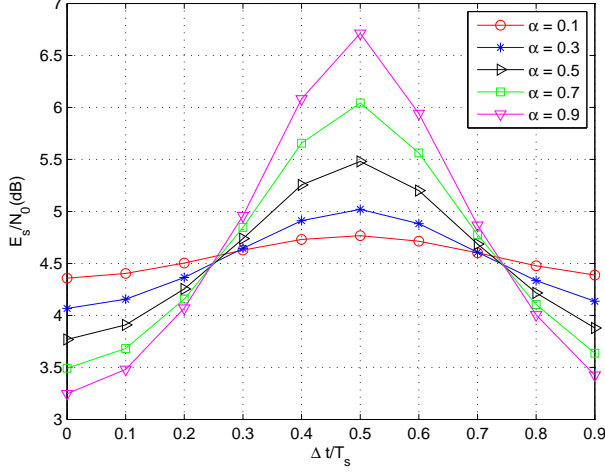
scenario of perfect synchronization, $H(e^{j\theta})$ stands for the Fourier transform of $h(k)$; in the scenario with timing errors, $H(e^{j\theta})$ represents the Fourier transform of $f(k)$.

From (12), it is obvious that the achievable information rate C is derived from E_s/N_0 considerations, and a certain E_s/N_0 is required to achieve a specified C. Let R be the target achievable information rate. In the perfect synchronization scenario, we have

$$C|_{H(e^{j\theta})=F[h(k)]} = R \qquad (13)$$

where $F\{\}$ represents the Fourier transform operator. Assume the needed E_s/N_0 in dBs at this point is SNR_h.

AS for the scenario with timing errors, let SNR_f be the E_s/N_0 in dBs at which

$$C|_{H(e^{j\theta})=F[f(k)]} = R \qquad (14)$$

Finally, the performance degradation L induced by timing mismatch is obtained as

$$L = SNR_f - SNR_h \qquad (15)$$

IV. NUMERICAL RESULTS AND DISCUSSIONS

In this section, Monte Carlo simulations based on the generalized expressions (12) and (15) are conducted to evaluate the effect of various system configurations on the performance degradation L induced by timing mismatch in DS-UWB systems. In the following simulations, in order to keep the simulation complexity on a reasonable level, the timing mismatch Δt on Rake diversity branches is set as $(0, 0.1, 0.2, ...0.9) \times T_s$, where T_s denotes the duration of one symbol. In addition, the numerical results are averaged over the best 900 out of 1000 IEEE 802.15.3a CM1 channel realizations, following the recommended instructions in [11] that the worst 10% channels are ignored in the simulation. The rest of this section consists of two parts: In the first part, the influence of timing mismatch on the performance of the widely used MRC S-Rake receiver is investigated in details under different system parameters; In the second part, we compare the performance loss of various Rake receivers, including MRC S-Rake, MRC P-Rake and EGC P-Rake, in the presence of mistiming.

The Influence of Timing Mismatch on MRC S-Rake Receivers: The needed E_s/N_0 curves for MRC S-Rake receivers in DS-UWB systems with different roll-off factors α are plotted in Fig.2. It is observed that if no timing mismatch exists, as α increases, the needed E_s/N_0 to achieve the target information Rate R gets smaller. However, the receiver with larger α is more sensitive to the timing mismatch Δt. As seen in this figure, when mistiming is small, the receiver with larger α needs less E_s/N_0 to obtain the target R; however, they requires more E_s/N_0 as mistiming aggravates.

Fig.3 depicts the E_s/N_0 needed for MRC S-Rake receivers with different diversity orders to achieve $R = 0.3$ when α equals to 1.0. As one can expect, when no timing errors exhibit, the MRC S-Rake receiver with more diversity branches needs less E_s/N_0 to achieve the target R. It is further seen that the sensitivity to timing mismatch increases with increasing

Fig. 2. The needed E_s/N_0 (dB) for MRC S-Rake receivers in DS-UWB systems with different roll-off factors α when $J = 8$ and $R = 0.3$
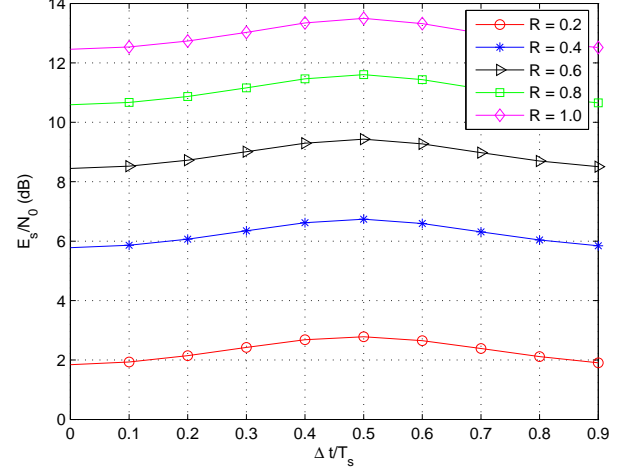

Fig. 4. The needed E_s/N_0 (dB) for MRC S-Rake receivers with different target achievable information rates R when $\alpha = 0.3$ and $J = 8$
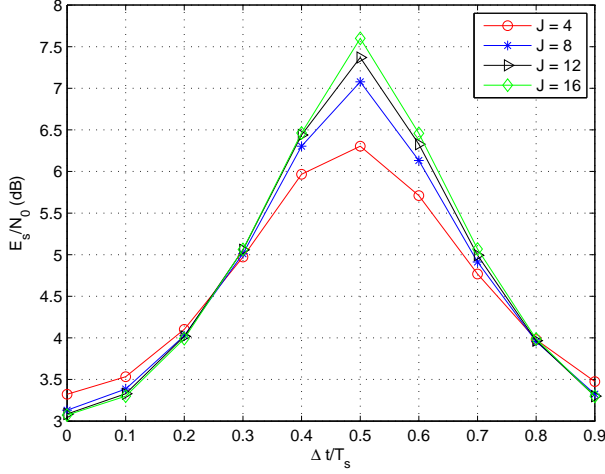

Fig. 3. The needed E_s/N_0 (dB) for MRC S-Rake receivers with different diversity orders J when $\alpha = 1.0$ and $R = 0.3$

Fig.5 - Fig.7 demonstrate the performance degradation of three types of Rake receivers, including MRC S-Rake, MRC P-Rake and EGC P-Rake, with respect to the variation of the excess bandwidth, the diversity order and the target achievable information rate under timing mismatch respectively. Fig.5 shows that with roll-off factor increasing, all types of Rake receivers obtain worse performance due to timing errors. Among all the receivers, MRC S-Rake is most sensitive to the increase of excess bandwidth and the EGC P-Rake is least sensitive. From Fig.6, it is observed that as the diversity order increases, the performance degradation of all the three kinds of Rake receivers gets larger. Fig.6 also shows that when MRC is employed, P-Rake is more sensitive to the change of Rake finger numbers than S-Rake, and in P-Rake receiver the EGC technique is more robust compared with MRC. Fig.7 demonstrates that the target information rate rarely impacts the performance loss of all the Rake receivers under timing mismatch, and the MRC S-Rake suffers the largest performance loss compared with other Rake receivers.

V. CONCLUSION

The effect of imperfect timing has been evaluated for the Rake reception in DS-UWB systems from the perspective of the achievable information rate. A generalized expression for the system performance degradation is derived, then corresponding simulations are conducted to investigate the effect of timing mismatch on the widely used MRC S-Rake receiver with respect to the excess bandwidth induced by the roll-off factor in RRC filters, the multipath diversity order and the target information rate. Simulation results illustrate that the performance loss has little relationship with the target information rate, but varies significantly with the system bandwidth and the diversity order, which further demonstrates that there exist fundamental trade-offs among the system timing requirement, the occupied bandwidth and the implementation complexity of DS-UWB systems. In addition, the performance
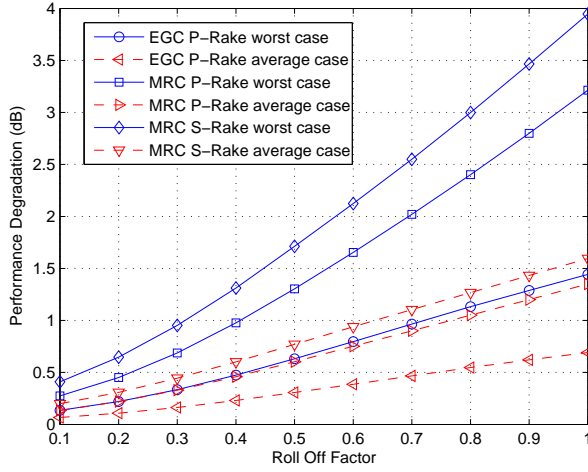
J in the MRC S-Rake receiver. Hence there also exists a design trade-off between the robustness to mistiming and the implementation complexity.

The needed E_s/N_0 for MRC S-Rake receivers to achieve various target information rates R is illustrated in Fig.4 when J is 8 and α equals to 0.3. It shows that as R increases, the needed E_s/N_0 increases obviously. However, the performance loss induced by timing mismatch rarely varies with the increase of the target achievable information rate in MRC S-Rake receivers.

The Comparison of Various Rake Receivers under Mistiming: In this part, the mistiming is assumed to follow uniform distribution [17]. We consider two cases: one is the worst case with the maximum performance degradation; the other is the average case, which represents the average degradation over the set of timing mismatch Δt.

Fig. 5. Performance degradation L (dB) as a function of the roll-off factor α for various Rake receivers when $J = 8$ and $R = 0.3$
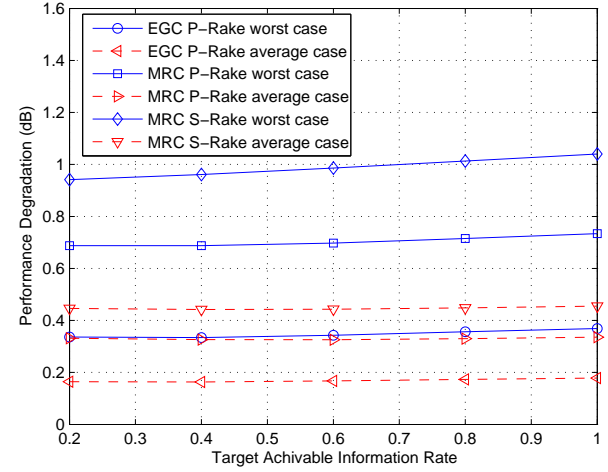

Fig. 7. Performance degradation L (dB) as a function of the target achievable information rate R for various Rake receivers when $\alpha = 0.3$ and $J = 8$
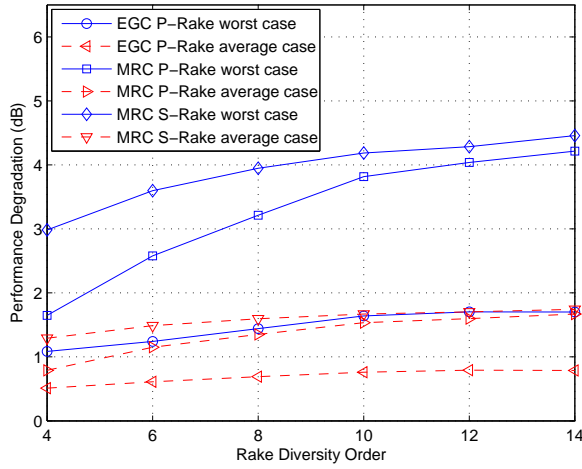

Fig. 6. Performance degradation L (dB) as a function of the diversity order J in various Rake receivers when $\alpha = 1.0$ and $R = 0.3$

degradation of various types of Rake receivers, including MRC S-Rake, MRC P-Rake and EGC P-Rake, is compared, and the sensitivity of different multipath component selection schemes and diversity combining techniques to the variation of system parameters are obtained. The numerical results also show that among the three types of Rake receivers, the MRC S-Rake suffers the most performance degradation in the presence of mistiming.

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